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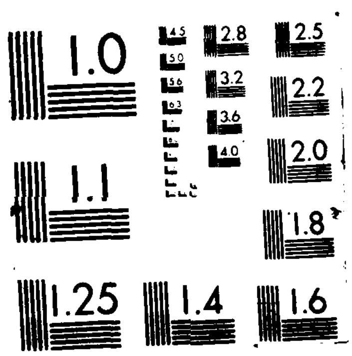
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Series Representations of Infinitely Divisible Random Vectors and a Generalized Shot Noise in Banach Spaces[1]

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ABSTRACT: A generalized shot noise in Banach spaces is defined as the a.s. limit of certain centered sums of dependent random vectors; and, a necessary and sufficient condition for its existence is given. As an immediate application, the LePage-type series representations of infinitely divisible random vectors are obtained.

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I. Introduction. In this paper we study the convergence and limit distribution of the centered sums

$$\sum_{j=1}^{n} H(\gamma_j, \xi_j) - A_n , \tag{1.1}$$

in connection with series representations of infinitely divisible random vectors. Here $\{\gamma_j\}$ is a sequence of arrivial times in a Poisson process, $\{\xi_j\}$ is a sequence of i.i.d. random elements, which is independent of $\{\gamma_j\}$, and H is a Banach space valued function.

Series representations involving arrival times in a Poisson process have been given by Ferguson and Klass [4], for real independent increment processes without Gaussian components. Kallenberg [8] showed the uniform convergence in the Ferguson-Klass decomposition and Resnick [18] related the decomposition to the well-known Itô-Lévy representation of processes with independent increments. A series representation of Hilbert space valued stable random vectors, that generalizes the Ferguson-Klass representation of one-dimensional stable random variables, has been established by LePage, Woodroofe and Zinn [12]. LePage [10] observed that symmetric stable random vectors can be represented as conditionally Gaussian. This important property has been generalized and extensively used by Marcus and Pisier [15] in their investigation of continuity of stable processes. Marcus and Pisier's work [15] showed the significance of the series decompositions in the study of stable probability measures on general Banach spaces (see also [5], [2], [19] and [21]). We refer the reader to [15] for a rigorous proof of the representation of symmetric stable vectors with values in arbitrary Banach spaces. A generalization of the one-dimensional Ferguson-Klass representation to the case of random vectors taking values in Banach spaces of cotype 2 is due to LePage [11]. Since this assumption on

the geometry of Banach spaces is too restrictive for many interesting applications of the representation (e.g. for studying the continuity of stochastic processes), it is necessary to investigate series developments without any restrictions on the Banach spaces. The validity of the LePage representation for certain symmetric infinitely divisible random vectors in general Banach spaces was stated by Marcus [14] (techniques similar to those of [15] can be used in that case, the general non-symmetric distributions considered here require different methods).

The main goal of the present paper is to give a simple and general scheme of deriving series representations of arbitrary Banach space valued infinitely divisible random vectors. Our approach uses an idea of Vervaat [22] who obtained the Ferguson-Klass decomposition of positive random variables as a particular case of a shot noise (for more information about shot noise see [22] and references therein). Since only a very restricted subclass of infinitely divisible probability measures can be represented by means of a shot noise (see Corollary 4.3(iii)), we introduce and study a *generalized shot noise*, which is defined as the a.s. limit of the centered sums (1.1). We obtain a full characterization of the convergence to a generalized shot noise in Section 2. In Section 3 we discuss certain special cases of the generalized shot noise and resulting simplifications in the centeres A_n. The results of Section 2 and 3 are applied to derive series representations of infinitely divisible random vectors in Section 4. This approach enables us to obtain various series representations, which generalize those of LePage [11], in a unified way, while avoiding many obscuring details due to specific forms of the function H is concrete situations.

Finally we would like to mention something about the methods in this paper. To determine the convergence in (1.1) we use a slight modification of the technique previously employed by Ferguson-Klass [4] who transformed certain dependent summand series into independent ones. The modification is that we associate with (1.1) a continuous time, independent increment, stochastic process, instead of the discrete time one, so that (1.1) is obtained by a random time substitution. This approach gives the results on the L^p-convergence immediately (see Corollary 2.5), and reveals a martingale structure of the decomposition (see Corollary 4.3(iv) and Theorem 3.1).

2. The convergence and distribution of a generalized shot noise.

We recall and complete some notation that will be used throughout the paper. $\{\xi_j\}_{j=1}^{\infty}$ is a sequence of i.i.d. random elements taking values in a measurable space $(D, \mathscr{D})$, with the common distribution $\mathscr{L}(\xi_j) = \lambda$. By $\{N(t)\}_{t\geq 0}$ is denoted a Poisson process with parameter 1 and γ_j is the jth arrival time of $N(t)$, i.e. $\gamma_j = \inf\{t > 0: N(t) = j\}$, $j = 1, 2, \ldots$. $\{U_j\}_{j=1}^{\infty}$ stands for a sequence of i.i.d. uniform on $(0, 1)$ random variables. We assume that $\{\xi_j\}_{j=1}^{\infty}$, $\{N(t)\}_{t\geq 0}$ and $\{U_j\}_{j=1}^{\infty}$ are defined on the same probability space $(\Omega, \mathscr{F}, P)$ and they are mutually independent.

In order to use the method of Ferguson and Klass [4] mentioned in the Introduction we shall need the following lemma which in the case $\mathscr{X} = \mathbb{R}$ can be deduced from Lemma 2[4] and then easily extended to the case when $\mathscr{X}$ is a separable Banach space. Since this lemma constitutes the first important step of the method and also may be of independent interest, we shall give below a straightforward and different proof in a more general case.

<u>LEMMA 2.1</u>. *Let* $(\mathscr{X}, \mathscr{B})$ *be a measurable vector space and let* $G: (0, \infty) \times D \to \mathscr{X}$ *be a measurable map. Then the* $\mathscr{X}$*-valued stochastic process given by*

$$X(t) = \sum_{j=1}^{N(t)} G(\gamma_j, \xi_j) \quad , \ t \geq 0 ,$$

has independent increments and

$$\mathscr{L}(X(t+s) - X(s)) = \mathscr{L}\Big(\sum_{j=1}^{N(t)} G(s + tU_j, \xi_j)\Big) .$$

Proof. Let $\mathcal{F}_t^{(1)} = \sigma(N(s): s \leq t)$ and $\mathcal{F}_k^{(2)} = \sigma(\xi_1, \ldots, \xi_k)$. Put

$$(2.1) \qquad \mathcal{F}_t = \{A \in \mathcal{F}: A \cap \{N(t) \leq k\} \in \mathcal{F}_t^{(1)} \vee \mathcal{F}_k^{(2)} \text{ for every } k \geq 1\} .$$

Then $\{\mathcal{F}_t\}_{t \geq 0}$ is an increasing filtration and $\{X(t)\}_{t \geq 0}$ is adapted to this filtration.

In order to prove that $\{X(t)\}_{t \geq 0}$ has independent increments it is enough to show that $\sigma(X(t+s) - X(s))$ and $\mathcal{F}_s$ are independent for every $t, s \geq 0$. Let $A \in \mathcal{F}_s$ and $B \in \mathcal{B}$. We get

$$P\{X(t+s) - X(s) \in B, A\} =$$

$$(2.2) \qquad \sum_{i,k \geq 0} P\{X(t+s) - X(s) \in B, N(s) = i, N(s+t) = i+k, A\} =$$

$$\sum_{i,k \geq 0} P\{\sum_{j=i+1}^{i+k} G(\gamma_j, \xi_j) \in B, N(t+s) - N(s) = k, A_i\} ,$$

where $A_i = \{N(s) = i, A\} \in \mathcal{F}_s^{(1)} \vee \mathcal{F}_i^{(2)}$ by (2.1). Since

$$\sum_{j=i+1}^{i+k} G(\gamma_j, \xi_j) = \sum_{j=i+1}^{i+k} G(s + \gamma_{j-i}^{(1)}, \xi_j) ,$$

where $\gamma_m^{(1)}$ is the m^{th} arrival time in the Poisson process $N^{(1)}(u) = N(u+s) - N(s)$, $u \geq 0$, we conclude that the events A_i and $\{\sum_{j=i+1}^{i+k} G(\gamma_j, \xi_j) \in B, N(t+s) - N(s) = k\}$ are independent. Therefore the last expression in (2.2) is equal to

$$\sum_{i,k \geq 0} P\{\sum_{j=i+1}^{i+k} G(s+\gamma_{j-i}^{(1)}, \xi_j) \in B, N^{(1)}(t) = k\} P(A_i) =$$

$$\sum_{i,k\geq 0} P\{\sum_{m=1}^{k} G(s+\gamma_m^{(1)}, \xi_m) \in B , N^{(1)}(t) = k\} P(A_i) =$$

$$\sum_{k\geq 0} P\{\sum_{j=1}^{k} G(s+\gamma_j,\xi_j) \in B, N(t) = k\}P(A) = P\{\sum_{j=1}^{N(t)} G(s+\gamma_j,\xi_j) \in B\}P(A) ,$$

which proves the independence of $\sigma(X(t+s) - X(s))$ and $\mathcal{F}_s$ as well as the equality $\mathcal{L}(X(t+s) - X(s)) = \mathcal{L}(\sum_{j=1}^{N(t)} G(s+\gamma_j, \xi_j))$.

In the proof of the second part of the lemma we shall use the well-known fact that the condtional distribution of $(\gamma_1, \ldots, \gamma_{N(t)})$ given that $N(t) = k \geq 1$ is equal to the distribution of $(tU_{(1)}, \ldots, tU_{(k)})$, where $U_{(j)}$ is the j^{th} order statistic of $U_1, \ldots, U_k$. We have, for every $B \in \mathcal{B}$,

$$P\{X(t+s) - X(s) \in B\} = P\{\sum_{j=1}^{N(t)} G(s+\gamma_j, \xi_j) \in B\} =$$

$$\sum_{k=0}^{\infty} P\{\sum_{j=1}^{k} G(s+tU_{(j)}, \xi_j) \in B\} \frac{t^k}{k!} e^{-t} =$$

$$\sum_{k=0}^{\infty} P\{\sum_{j=1}^{k} G(s+tU_j, \xi_j) \in B\} \frac{t^k}{k!} e^{-t} =$$

$$P\{\sum_{j=1}^{N(t)} G(s+tU_j, \xi_j) \in B\} ,$$

which completes the proof.

<u>LEMMA 2.2</u>. *Under the notations of Lemma 2.1, if* $(\mathfrak{X}, \mathcal{A}) = (\mathbb{R}, \mathcal{B}_{\mathbb{R}})$, *then*

(i) $EX(t) = \int_0^t \int_D G(u, v)\lambda(dv)du$,

provided either one of the above quantities, on the left or right side, exists;

(ii) $E \exp[iX(t)] = \exp\{\int_0^t \int [e^{iG(u,v)}-1]\lambda(dv)du\}$.

Proof. By Lemma 2.1 we get

$$E\ X(t) = E[\sum_{j=1}^{N(t)} G(tU_j,\ \xi_j)] =$$

$$\sum_{k\geq 0} E[\sum_{j=1}^{k} G(tU_j,\ \xi_j)I(N(t) = k)] =$$

$$\sum_{k\geq 0} k\ E[G(tU_1,\ \xi_1)]\ \frac{t^k}{k!}\ e^{-t} =$$

$$tEG(tU_1,\ \xi_1) = t\int_0^1 \int_D G(ts,\ v)\lambda(dv)ds =$$

$$\int_0^t \int_D G(u,\ v)\lambda(dv)du\ ,$$

which gives (i). The proof of (ii) is similar.

The method of random time substitutuon will require the existence of the limit as $t \to \infty$ for almost every sample path of the associated stochastic process. The next lemma will be useful for this purpose. Its proof is routine and will be omitted.

LEMMA 2.3. *Let* $\{Y(t)\}_{t\geq 0}$ *be a stochastic process with values in a separable metric space and whose sample paths are right-continuous. Then* $\lim_{t\to\infty} Y(t,\ \omega)$ *exists for a.e.* ω *if and only if for every increasing sequence* $\{t_n\}_{n=1}^{\infty}$ *with* $\lim_{n\to\infty} t_n = \infty$, *the sequence* $\{Y(t_n)\}_{n=1}^{\infty}$ *converges a.s..*

To state and prove the main result of this section we shall need some notation that will be also used throughout this paper. E will stand for a separable Banach space with the norm $\|\cdot\|$ and $B_r = \{x \in E: \|x\| \leq r\}$, $r \geq 0 (B_\infty = E)$. The dual of E will be denoted by E' and $\langle x', x\rangle \equiv x'(x)$, $x' \in E'$, $x \in E$.

We recall that a measure M on $\mathcal{B}_E$ with $M(\{0\}) = 0$ is said to be a Lévy measure if for every $x' \in E'$, $\int_E (\langle x', x\rangle^2 \wedge 1) M(dx) < \infty$ and for some (each) $r \in (0, \infty)$ the function ϕ_r defined by

$$\phi_r(x') = \exp\{\int_E [e^{i\langle x', x\rangle} - 1 - i\langle x', x\rangle I_{B_r}(x)] M(dx)\},$$

$x' \in E'$, is characteristic function of a probability measure on E. The probability measure with characteristic function ϕ_r will be denoted by $c_r\mathrm{Pois}(M)$ (see: deAcosta et al. [1]). If M is a Lévy measure and additionally $\int_{B_1^c} \|x\| M(dx) < \infty$ ($\int_{B_1} \|x\| M(dx) < \infty$, respectively), then we define $c_\infty\mathrm{Pois}(M)$ ($c_0\mathrm{Pois}(M)$, respectively) as a probability measure with characteristic function ϕ_∞ (ϕ_0, respectively).

Let $H: (0, \infty) \times D \to E$ be a Borel measureable map and define a measure F on $\mathcal{B}_E$ by

$$(2.5) \qquad F(A) = \int_0^\infty \int_D I_{A\setminus\{0\}}(H(u, v)) \lambda(dv) du \quad , A \in \mathcal{B}_E .$$

Note that $F(\{0\}) = 0$. Put

$$A(t) = \int_0^t \int_D H(u, v) I_{B_1}(H(u, v)) \lambda(dv) du, \quad t \geq 0 .$$

THEOREM 2.4. *Let* $T_n = \sum_{j=1}^{n} H(\gamma_j, \xi_j) - A(\gamma_n)$. *Then* $\{T_n\}$ *converges a.s. in the norm of* E *if and only if* F *is a Lévy measure on* E . *Further, if* F *is a Lévy measure and* $T_\infty = \lim_{n\to\infty} T_n$, *then*

$$\mathcal{L}(T_\infty) = c_1 \mathrm{Pois}(F) .$$

Proof. Let

$$X(t) = \sum_{j=1}^{N(t)} H(\gamma_j, \xi_j) - A(t) , \quad t \geq 0 .$$

By Lemma 2.1 $\{X(t)\}_{t\geq 0}$ is an independent increment E-valued stochastic process with right-continuous sample paths. Using Lemma 2.2(ii) we get

$$(2.6) \qquad \mathcal{L}(X(t)) = c_1 \mathrm{Pois}(F^{(t)}) ,$$

where

$$(2.7) \qquad F^{(t)}(A) = \int_0^t \int_D I_{A\setminus\{0\}}(H(u, v))\lambda(dv)du \quad , A \in \mathcal{B}_E ,$$

(note that $F^{(t)}(E) = t < \infty$) .

Assume first that F is a Lévy measure. Since $F^{(t)} \nearrow F$ as $t \nearrow \infty$, we get

$$c_1 \mathrm{Pois}(F^{(t)}) \Rightarrow c_1 \mathrm{Pois}(F) \quad \text{as } t \nearrow \infty$$

(see deAcosta et al. Theorem 1.6). Hence, by Itô-Nisio Theorem ([7], Theorem 1) and (2.6), $\{X(t_n)\}_{n=1}^{\infty}$ converges a.s. for each $t_1 < t_2 < \ldots < t_n \to \infty$. In view of Lemma 2.3 $X = \lim_{t\to\infty} X(t)$ exists a.s. Clearly, $\mathcal{L}(X) = c_1 \mathrm{Pois}(F)$. Now we notice

that $T_n = X(\gamma_n)$ and $\gamma_n \to \infty$ a.s. Therefore $T_n \to T_\infty \equiv X$ a.s. as $n \to \infty$, which ends the proof of the sufficency part of the theorem.

Now we prove the necessity. Assume that $\{T_n\}$ converges a.s. We have, for every t,

(2.8) $$T_{N(t)+1} = X(t) + Y(t) ,$$

where

$$Y(t) = H(\gamma_{N(t)+1}, \xi_{N(t)+1}) + A(t) - A(\gamma_{N(t)+1}) .$$

By Markov property of $\{N(s)\}_{s\geq 0}$, the random vectors $X(t)$ and $Y(t)$ are independent for each t. Since $T_{N(t)+1} \to T_\infty$ a.s. as $t \to \infty$, by (2.8) $\{\mathcal{L}(X(t))\}_{t\geq 0}$ is relatively shift compact. In view of (2.6) and Theorem 1.6 in [1] F is a Lévy measure. The proof is complete.

COROLLARY 2.5. *Let* F *be a Lévy measure and* $\int_{B_1^c} \|x\|^p F(dx) < \infty$ *for some* $0 < p < \infty$. *Then* $T_n \to T_\infty$ *a.s. and in* L_E^p.

Proof. Since $E\|X\|^p < \infty$, $E \sup_{0\leq t<\infty} \|X(t)\|^p < \infty$ by Corollary 3.3 in Hoffmann-Jørgensen [6]. Hence

$$E \sup_n \|T_n\|^p = E \sup_n \|X(\gamma_n)\|^p \leq E \sup_{0\leq t<\infty} \|X(t)\|^p < \infty ,$$

which ends the proof. □

REMARK 2.6. Theorem 2.4, when specified to those Banach spaces for which a full characterization of Lévy measures is known, gives definitive conditions in terms of the function H for the a.s. convergence of $\{T_n\}$. For example, if

$E = R^n$ or more general, if E is a separable Hilbert space, then $\int_0^\infty \int_D (1 \wedge \|H(u, v)\|^2)\lambda(dv)du < \infty$ is neccessary and sufficient for the a.s. convegence of $\{T_n\}$. Similarly, if $E = \ell^p$, $2 \leq p < \infty$, the conjunction of the following two conditions is equivalent to the a.s. convergence of $\{T_n\}$:

$$\int_0^\infty \int_D (1 \wedge \|H(u, v)\|^p)\lambda(dv)du < \infty$$

and

$$\sum_{j=1}^\infty \left[\int_0^\infty \int_D |\langle H(u, v), e_j\rangle|^2 I_{B_1}(H(u, v))\lambda(dv)du\right]^{p/2} < \infty ,$$

where $\{e_j\}$ denotes the standard basis (see [13], p. 75).

3. Convergence in some special cases.

In this section we shall discuss some interesting modifications in (1.1) which are possible when F satisfies certain additional hypotheses.

THERE0REM 3.1. *Assume that* F , *defined by* (2.5), *is a Lévy measure on* E *such that* $\int_{B_1^c} \|x\|^p F(dx) < \infty$ *for some* $p \geq 1$. *Let*

$$C(t) = \int_0^t \int_D H(u, v)\lambda(dv)du \quad , \ t \geq 0 .$$

Then

(i) $M_n = \sum_{j=1}^{n} H(\gamma_j, \xi_j) - C(\gamma_n)$, $n \geq 1$, *is a martingale with respect to* $\sigma(\gamma_1, \ldots, \gamma_n, \xi_1, \ldots, \xi_n)$,

(ii) $M_n \to M_\infty$ *a.s. and in* L_E^p *as* $n \to \infty$,

(iii) $\mathcal{L}(M_\infty) = c_\infty \mathrm{Pois}(F)$.

Proof. First note that $C(t)$ is well-defined as a Bochner integral. Indeed,

$$\int_0^t \int_D \|H(u, v)\| \lambda(dv)du \leq t + \int_0^t \int_D \|H(u, v)\| I_{B_1^c} (H(u, v))\lambda(dv)du$$

$$\leq t + \int_{B_1^c} \|x\|^p F(dx) < \infty .$$

Put $X_1(t) = \sum_{j=1}^{N(t)} H(\gamma_j, \xi_j) - C(t) = X(t) + A(t) - C(t)$, where $X(t)$ is defined in the proof of Theorem 2.4. In the proofs of Theorem 2.4 and Corollary 2.5 we have shown that

$$X(t) \to X \quad \text{a.s.} \quad \text{as} \quad t \to \infty ,$$

$$\mathcal{L}(X) = c_1 \mathrm{Pois}(F) \quad \text{and}$$

$$E \sup_{0 \le t < \infty} \|X(t)\|^p < \infty \quad .$$

Since

$$A(t) - C(t) = - \int_0^t \int_D H(u, v) I_{B_1^c}(H(u, v))\lambda(dv)du \to -\int_{B_1^c} x \, dF(x) ,$$

as $t \to \infty$, we conclude that

$$X_1(t) \to X_1 \quad \text{a.s.,} \quad \text{as} \quad t \to \infty ,$$

$$\mathcal{L}(X_1) = c_\infty \mathrm{Pois}(F) \quad \text{and}$$

(3.1) $$E \sup_{0 \le t < \infty} \|X_1(t)\|^p < \infty .$$

By Lemmas 2.1 and 2.2, $\{X_1(t)\}_{t \ge 0}$ is an independent increment process with right continuous sample paths and $EX_1(t) = 0$. Moreover, $\{X_1(t)\}_{t \ge 0}$ is adapted to the filtration $\{\mathcal{F}_t\}_{t \ge 0}$ defined by (2.1) and $X_1(t+s) - X_1(s)$ is independent of $\mathcal{F}_s$. Hence $\{X_1(t), \mathcal{F}_t\}_{t \ge 0}$ is a martingale. By (3.1) and the Optional Sampling Theorem

$$M_n = X_1(\gamma_n) , \quad n \ge 1 ,$$

form a martingale with respect to $\mathcal{F}_{\gamma_n} \supset \sigma(\gamma_1, \ldots, \gamma_n, \xi_1, \ldots, \xi_n)$ and clearly $M_n \to M_\infty \equiv X_1$ a.s. and in L^p_E. The proof is complete.

THEOREM 3.2. *Assume that* F, *defined by* (2.5), *is a Lévy measure such that* $\int_{B_1} \|x\| F(dx) < \infty$. *Then*

$$S_n = \sum_1^n H(\gamma_j, \xi_j) \to S_\infty \quad a.s., \ as \quad n \to \infty ,$$

and

$$\mathcal{L}(S_\infty) = c_0 \mathrm{Pois}(F) .$$

Proof. Since

$$\int_0^\infty \int_D \|H(u, v)\| I_{B_1}(H(u, v)) \lambda(dv) du = \int_{B_1} \|x\| F(dx) < \infty ,$$

it follows by the Dominated Convergence Theorem that

$$A(\gamma_n) \to \int_{B_1} x \, F(dx) \quad \text{a.s.,} \quad \text{as} \quad n \to \infty .$$

An appeal to Theorem 2.4 completes the proof.

The other case when the centering in (1.1) is not needed occurs when F is symmetric. From now on $\{\varepsilon_j\}_{j=1}^\infty$ will be a sequence of i.i.d. random variables such that $P\{\varepsilon_j = 1\} = 1 - P\{\varepsilon_j = -1\} = \frac{1}{2}$. Further, we assume that $\{\varepsilon_j\}$, $\{\gamma_j\}$ and $\{\xi_j\}$ are independent of each other.

<u>THEOREM 3.3</u>. *Assume that* F , *defined by* (2.5), *is a symmetric Lévy measure on* E . *Then*

$$\tilde{S}_n \equiv \sum_{j=1}^{n} \varepsilon_j H(\gamma_j, \xi_j) \to \tilde{S}_\infty \quad a.s., \text{ as } \quad n \to \infty ,$$

and

$$\mathcal{L}(\tilde{S}_\infty) = c_1 \operatorname{Pois}(F) .$$

<u>*Proof*</u>. We can write

$$\tilde{S}_n = \sum_{j=1}^{n} \tilde{H}(\gamma_j, \tilde{\xi}_j) ,$$

where $\tilde{\xi}_j = (\varepsilon_j, \xi_j)$ takes values in $\{-1, 1\} \times D$, $\tilde{H}(u, \tilde{v}) = v_1 H(u, v_2)$, $u \geq 0$, $\tilde{v} = (v_1, v_2) \in \{-1, 1\} \times D$. We have

$$\tilde{\lambda} \equiv \mathcal{L}(\tilde{\xi}_j) = (\tfrac{1}{2}\, \delta_{-1} + \tfrac{1}{2}\, \delta_1) \times \lambda .$$

Thus

$$\tilde{F}(A) = \int_0^\infty \int_{\{-1,1\}\times D} I_{A\setminus\{0\}}(\tilde{H}(u, \tilde{v}))\tilde{\lambda}(d\tilde{v})du =$$

$$\frac{1}{2} F(-A) + \frac{1}{2} F(A) = F(A) , \quad A \in B_E ,$$

and

$$\tilde{A}(t) = \int_0^t \int_{\{-1,1\}\times D} \tilde{H}(u, \tilde{v}) I_{B_1}(\tilde{H}(u, \tilde{v}))\tilde{\lambda}(d\tilde{v})du = 0 \quad ,$$

for every $t \geq 0$. Theorem 2.4 completes the proof.

Random centers $A_n = A(\gamma_n)$ in (1.1) provide a fine connection between the centered sums and the associated compound Poisson process. Random centers $A_n = C(\gamma_n)$ are also necessary for the martingale property in Theorem 3.1. Nevertheless, it is an interesting question whether random centers can be replaced by non-random ones and the a.s. convergence still would hold? We could not answer this question in its full generality but under certain additional conditions the answer is yes. To procede this question we begin with a lemma that is a special case of Lemma 4 in Klass and Ferguson [4]. We shall give below a short proof of this lemma and also indicate that our method can be easily extended to obtain a new and short proof of Lemma 4 in [4].

LEMMA 3.4. *Let* g *be a non-increasing square integrable function defined on* $(0, \infty)$. *Then*

$$\int_n^{\gamma_n} g(u)du \to 0 \quad a.s., \text{ as } n \to \infty .$$

Proof. We have

$$(3.2) \qquad \left| \int_n^{\gamma_n} g(u)dt \right| \leq g(\gamma_n \wedge n) \, |\gamma_n - n| ,$$

by the monotonicity of g , further, by the Strong Law of Large Numbers we have with probability one:

$$(3.3) \qquad g(\gamma_n \wedge n) \leq g(\tfrac{n}{2}) \quad \text{eventually.}$$

Using Hajek-Renyi-Chow inequality [3] p. 243 we get, for every $\varepsilon > 0$,

$$P\{\max_{m\le k\le n} g(\tfrac{k}{2})\ |\gamma_k - k| \ge \varepsilon\} \le \varepsilon^{-2}\sum_{k=m}^{n} g^2(\tfrac{k}{2}) \to 0$$

as m , $n \to \infty$. Thus $g(\frac{n}{2})\ |\gamma_n - n| \to 0$ a.s., which combined with (3.2) and (3.3) completes the proof.

THEOREM 3.5. *Assume that* F , *defined by* (2.5), *is a Lévy measure on* E *such that* $\int_E (\|x\|^2 \wedge 1) F(dx) < \infty$. *Suppose that, for each* $v \in D$, $\|H(u, v)\|$ *is a non-increasing function of* $u \in (0, \infty)$. *Then*

$$\sum_{j=1}^{n} H(\gamma_j, \xi_j) - A(n) \to T_\infty \quad ,$$

where T_∞ *is specified in Theorem* 2.4.

Proof. Let

$$V_n = A(\gamma_n) - A(n) = \int_n^{\gamma_n} \int_D H(u,v) I_{B_1}(H(u,v))\lambda(dv)du$$

and

$$g(u) = \{\int_D (\|H(u,v)\|^2 \wedge 1)\lambda(dv)\}^{1/2} \quad .$$

g is non- increasing,

$$\int_0^\infty g^2(u)du = \int_E (\|x\|^2 \wedge 1)F(dx) < \infty ,$$

and we have

$$\|V_n\| \leq \left|\int_n^{\gamma_n} \int_D (\|H(u,v)\| \wedge 1)\lambda(dv)du\right|$$

$$\leq \left|\int_n^{\gamma_n} g(u)du\right|$$

by Jensen's inequality. Applying Lemma 3.4 we get $V_n \to 0$ a.s.. Theorem 2.4 completes the proof.

4. Series representations of infinitely divisible random vectors.

Let F be a Borel measure on E with $F(\{0\}) = 0$. We say that F admits a *polar decomposition with respect to a Borel set* D, $0 \notin D \subset E$, if

$$(4.1) \qquad F(A) = \int_D \int_{(0,\infty)} I_A(tx)\rho(x,dt)\lambda(dx)\ , \ A \in \mathcal{B}_E\ ,$$

where $\{\rho(x,\cdot)\}_{x \in D}$ is a measurable family of Borel measures on $(0, \infty)$ and λ is a Borel probability measure on D. The phrase "*polar decomposition*" will always mean a polar decomposition with respect to the unit sphere $D = S_1 = \{x \in E: \|x\| = 1\}$.

A polar decomposition of Lévy measures on Hilbert spaces and its application to stochastic integral representations of infinitely divisible processes were studied by Rajput and Rosinski [17]. We shall show here that also Lévy measures on general Banach spaces admit polar decompositions so that (4.1) can always be assumed. In fact, we shall prove more:

PROSITITION 4.1. *Let* M *be a Borel measure on* E *such that* $M(\{0\}) = 0$ *and* $M(B_r^c) < \infty$ *for every* $r > 0$. *Then* M *admits a polar decomposition.*

Proof. If $F \equiv 0$, then (4.1) holds trivially with $\rho(\cdot, \cdot) \equiv 0$ and an arbitrary λ. Therefore we may assume that $0 < F(E) \leq \infty$. We shall construct a Borel function $f: [0, \infty) \to [0, \infty)$ that vanishes only at 0 and satisfies

$$\int_E f(\|x\|)F(dx) = 1\ .$$

Let $r_0 = \inf\{r: M(B_r^c) = 0\}$, $0 < r_0 \leq \infty$. Define

$$\phi(t) = \begin{cases} [e^t M(B_r^c)]^{-1} & \text{if } 0 < t < r_0 \\ 0 & \text{otherwise.} \end{cases}$$

Put $f(u) = (1 - e^{-r_0})^{-1} \int_0^u \phi(t)dt$. f vanishes only at 0 and

$$\int_E f(\|x\|)M(dx) = (1 - e^{-r_0})^{-1} \int_E \int_0^{\|x\|} \phi(t)dtM(dx) =$$

$$(1 - e^{-r_0})^{-1} \int_0^\infty \int_E \phi(t) I_{(0,\|x\|)}(t) M(dx)dt =$$

$$(1 - e^{-r_0})^{-1} \int_0^{r_0} e^{-t}dt = 1 .$$

Define now a probability measure G on E by $G(dx) = f(\|x\|)M(dx)$. Since $G(\{0\}) = 0$, $G_0 \circ \psi^{-1}$ is a probability measure on $S_1 \times (0, \infty)$, where $G_0 = G_{|E\setminus\{0\}}$, and ψ: $E\setminus\{0\} \to S_1 \times (0, \infty)$ is defined by $\psi(x) = (\frac{x}{\|x\|}, \|x\|)$. Let λ be the marginal distribution of $G_0 \circ \psi^{-1}$ given by

$$\lambda(B) = (G_0 \circ \psi^{-1})(B \times (0, \infty)) , \quad B \in \mathcal{B}_{S_1} .$$

Using the well-known fact on the existence of regular conditional probabilities we get that there exists a measurable family $\{\nu(x, \cdot)\}_{x \in S_1}$ of probability measures on $(0, \infty)$ such that, for every $C \in \mathcal{B}_{S_1 \times (0, \infty)}$

$$(G_0 \circ \psi^{-1})(C) = \int_{S_1} \int_{(0,\infty)} I_C(x,t)\nu(x,dt)\lambda(dx) .$$

Hence, for every $A \in \mathcal{B}_E$

$$G(A) = G_0(A\setminus\{0\}) = \int_{S_1}\int_{(0,\infty)} I_A(tx)\nu(x,dt)\lambda(dx)$$

which yields

$$F(A) = \int_A \frac{1}{f(\|x\|)}\, G(dx) =$$

$$\int_{S_1}\int_{(0,\infty)} \frac{1}{f(\|x\|)}\, I_A(tx)\nu(x,dt)\lambda(dx) =$$

$$\int_{S_1}\int_{(0,\infty)} I_A(tx)\, \frac{\nu(x,dt)}{f(t)}\, \lambda(dx) \quad .$$

Therefore (4.1) is fulfilled with $\rho(x, dt) = \dfrac{\nu(x, dt)}{f(t)}$.

PROPOSITION 4.2. *Let* F *be a Borel measure on* E *satisfying* (4.1). *Let, for each* $v \in D$,

(4.2) $$R(u, v) \equiv \inf\{t > 0: \rho(v, (t, \infty)) \le u\} \ , \ u > 0 \ ,$$

be the right continuous inverse of the function $t \to \rho(v, (t, \infty))$. *Then the function* H *defined by*

$$H(u, v) \equiv R(u, v)v$$

satisfies (2.5).

Proof. For every $A \in \mathcal{B}_E$ we have

$$\int_0^\infty \int_D I_{A\setminus\{0\}} (R(u, v)v)\lambda(dv)du =$$

$$\int_D [\int_0^\infty I_{A\setminus\{0\}} (R(u, v)v)du]\lambda(dv) =$$

$$\int_D [\int_0^\infty I_{A\setminus\{0\}}(tv)\rho(v, dt)]\lambda(dv) = F(A) ,$$

where we utilized the fact that $\mathrm{Leb}(\{u \geq 0: R(u, v) \in (t, \infty)\}) = \rho(v, (t, \infty))$, $t \geq 0$.

The results of sections 2 and 3 when specified to the case $H(u, v) = R(u, v)v$ give the following generalizations of the LePage's result ([11], Theorem 2).

<u>COROLLARY 4.3</u>. *Let* μ *be an infinitely divisible probability measure on* E *without Gaussian component i.e.*

(4.3) $\quad \mu = \delta_a * c_1\mathrm{Pois}(F)$,

where $a \in E$ *and* F *is a Lévy measure. Assume that* F *admits a polar decomposition* (4.1) *and let* R *be defined by* (4.2). *Put* $S_n = \sum_{j=1}^{n} R(\gamma_j, \xi_j)\xi_j$ *and*

$$A(t) = \int_0^t \int_D R(u,v)vI_{B_1}(R(u,v)v)\lambda(dv)du , \quad t \geq 0 .$$

Then

(i) $S_n - A(\gamma_n)$ *converges a.s., as* $n \to \infty$, *and* $\mathcal{L}(\lim[S_n - A(\gamma_n) + a]) = \mu$. *If* $\int_E \|x\|^p \mu(dx) < \infty$ *for some* $p > 0$, *then the convergence holds also in the* L^p_E *norm.*

(ii) *If* $\int_E (\|x\|^2 \wedge 1)\, F(dx) < \infty$, *then* $S_n - A(n)$ *converges a.s., as* $n \to \infty$, *and* $\mathcal{L}(\lim[S_n - A(n) + a]) = \mu$.

(iii) *If* $\int_{B_1} \|x\|\, F(dx) < \infty$, *then* S_n *converges a.s., as* $n \to \infty$, *and* $\mathcal{L}(\lim S_n + a_0) = \mu$, *where* $a_0 = a - \int_{B_1} xF(dx)$. *In addition,* S_n *converges in* L_E^p *provided* $\int_E \|x\|^p\, \mu(dx) < \infty$ *for some* $p > 0$.

(iv) *If* $\int_E \|x\|^p \mu(dx) < \infty$ *for some* $p \geq 1$, *then* $M_n = S_n - C(\gamma_n)$ *is a martingale with respect to* $\sigma(\gamma_1, \ldots, \gamma_n, \xi_1, \ldots, \xi_n)$, M_n *converges a.s. and in* L_E^p, *as* $n \to \infty$, *and* $\mathcal{L}(\lim M_n + a_1) = \mu$, *where* $a_1 = a + \int_{B_1^c} xF(dx)$ *and*

$$C(t) = \int_0^t \int_D R(u, v)\, v\lambda(dv)\, du\ , \quad t \geq 0 .$$

(v) *If* μ *is symmetric, then* $\tilde{S}_n = \sum_{j=1}^{n} \varepsilon_j R(\gamma_j, \xi_j)\xi_j$ *converges a.s. as* $n \to \infty$ *and* $\mathcal{L}(\lim \tilde{S}_n) = \mu$. *In addition,* $\tilde{S}_n$ *converges in* L_E^p *provided* $\int_E \|x\|^p\, \mu(dx) < \infty$, *for some* $p > 0$.

Proof. Indeed, by Proposition 4.2 the equality (2.5) is satisfied. Thus, (i) follows from Theorem 2.4 and Corollary 2.5; (ii) is a consequence of Theorem 3.5; Theorem 3.2 justifies the first part of (iii) and the second part follows from Corollary 2.5 and the observation that $\|A(\gamma_n)\|$ is uniformly bounded by $\int_{B_1} \|x\|\, F(dx)$; (iv) is a corollary to Theorem 3.1; (v) follows from Theorem 3.3 and Corollary 2.5. The proof is complete.

A few comments are now in order. First note that Corollary 4.3(i) and (ii) generalize LePage's Theorem 2 [11] by removing the restriction concerning the geometry of Banach space E and in our case D is an arbitrary Borel set. This makes the representation useful in investigation, for example, general infinitely divisible processes with sample paths in arbitrary Banach spaces. The results on the

L_E^p -convergence and the martingale development given in (iv) are also new. Finally, we note that the centering constants in LePage [11], Theorem 2, are erroneous. They should be assumptotically equal to A(n).

The representation of μ becomes simpler when a polar decomposition of F is of product type for some D, i.e.

$$(4.4) \qquad F(A) = \int_D \int_{(0,\infty)} I_A(tx)\rho(dt)\lambda(dx)$$

for all $A \in \mathcal{B}_E$. In this case, $\rho(x, \cdot) \equiv \rho(\cdot)$ is the same Lévy measure for all x's .

LEMMA 4.4. *Let* F *be a Levy measure on* E *which satisfies* (4.4), *where* D *is bounded. Then* $\int_E (\|x\|^2 \wedge 1)F(dx) < \infty$.

Proof. Let $d = \sup\{\|x\|: x \in D\} < \infty$. We have

$$\int_E (\|x\|^2 \wedge 1)F(dx) = \int_D \int_{(0,\infty)} (\|tx\|^2 \wedge 1)\rho(dt)\lambda(dx)$$

$$\leq \int_{(0,\infty)} (d^2t^2 \wedge 1)\rho(dt) < \infty .$$

The above lemma and Corollary 4.3(ii) give the following

COROLLARY 4.5. *Let* μ *be given by* (4.3) *and let* F *admits decomposition* (4.4) *with* D *bounded. Define*

$$R(u) = \inf\{t > 0: \rho((t, \infty)) \leq u\} , \quad u > 0 ,$$

as the right-continuous inverse of the function $t \to \rho((t,\infty))$. *Then*

$$\sum_{j=1}^{n} R(\gamma_j)\xi_j - b_n + a \to T \quad a.s., \text{ as } \quad n \to \infty ,$$

and $\mathcal{L}(T) = \mu$, *where*

$$b_n = \int_0^n [R(u) \int_D v I_{B_1}(R(u)v)\lambda(dv)]du .$$

<u>EXAMPLE</u>: <u>General stable distributions</u>.

Let μ be a p-stable probability measure on E , $0 < p < 2$. In view of Lévy spectral representation theorem there exists a finite Borel measure σ on S_1 and $x_0 \in E$ such that the characteristic function $\hat{\mu}$ of μ can be written as follows:

$$\hat{\mu}(x') = \exp\{-\int_{S_1} |\langle x',x\rangle|^p \sigma(dx) + iQ_p(\sigma,x') + i\langle x',x_0\rangle\} , \tag{4.5}$$

where

$$Q_p(\sigma,x') = \begin{cases} \tan(\pi p/2) \int_{S_1} |\langle x',x\rangle|^p \operatorname{sign}\langle x',x\rangle \sigma(dx) , & p \neq 1 , \\ -2/\pi \int_{S_1} \langle x',x\rangle \ln|\langle x',x\rangle| \sigma(dx) , & p = 1 . \end{cases}$$

(for this and further facts concerning stable measures we refer the reader to Linde [13], Chapter 6.3). In order to obtain series representation of μ we write μ in the form (4.3). Elementary computations give

$$a = \begin{cases} x_0 - (c_p(p-1))^{-1}\sigma(S_1)\bar{x}_\sigma \quad , \quad p \neq 1 \\ \\ x_0 - 2(1-\gamma)/\pi\ \sigma(S_1)\bar{x}_\sigma \quad , \quad p = 1 \ , \end{cases}$$

where $c_p = \cos(\pi p/2)\Gamma(-p)$, $p \neq 1$, $c_1 = \pi/2$, γ denotes Euler's constant , and

$$\bar{x}_\sigma = \int_{S_1} x\sigma(dx)/\sigma(S_1) \ .$$

Further, we can represent the Lévy measure F of μ as follows:

$$F(A) = c_p^{-1} \int_{S_1} \int_{(0,\infty)} I_A(tx)t^{-1-p}dt\ \sigma(dx) =$$

$$\int_{S_1} \int_{(0,\infty)} I_A(tx)\rho(dt)\lambda(dx) \ ,$$

where $\rho(dt) = c_p^{-1}\ \sigma(S_1)t^{1-p}dt$, $\lambda(dx) = \sigma(dx)/\sigma(S_1)$. Therefore, the assumptions of Corollary 4.5 are satisfied, and we compute

$$R(u) = d_p\sigma^{1/p}(S_1)u^{-1/p} \ ,$$

where $d_p = (pc_p)^{-1/p}$, and, for $n \geq d_p^p\ \sigma(S_1)$,

$$b_n = \begin{cases} p/(p-1)[d_p\sigma^{1/p}(S_1)n^{1-1/p} - d_p^p\ \sigma(S_1)]\bar{x}_\sigma, \quad p \neq 1 \\ 2/\pi[\ln n - \ln(2/\pi\ \sigma(S_1))]\sigma(S_1)\bar{x}_\sigma \ , \ p = 1 \ . \end{cases}$$

Under the above notations, using Corollaries 4.5, 4.3(iii) and (iv), we obtain

COROLLARY 4.6. *Let* μ *be a p-stable probability measure on* E *with the characteristic function given by* (4.5), $0 < p < 2$. *Let*

$$V_n = d_p\sigma^{1/p}(S_1)\{\sum_{j=1}^{n}\gamma_j^{-1/p}\xi_j - k(n)\overline{x}_\sigma\} + x_0 ,$$

where

$$k(t) = \begin{cases} (1 - 1/p)^{-1}t^{1-1/p} & , 1 < p < 2 \\ \ln t + 1 - \gamma - \ln(d_1\sigma(S_1)), & p = 1 \\ 0 & , 0 < p < 1 . \end{cases}$$

Then $V = \lim_{n\to\infty} V_n$ *exists a.s. and* $\mathcal{L}(V) = \mu$. *Further, for* $1 < p < 2$, *put*

$$M_n = d_p\sigma^{1/p}(S_1)\{\sum_{j=1}^{n}\gamma_j^{-1/p}\xi_j - k(\gamma_n)\overline{x}_\sigma\} + x_0 .$$

Then M_n *is a martingale with respect to* $\sigma(\gamma_1,\ldots,\gamma_n,\xi_1,\ldots,\xi_n)$, $n \geq 1$, $M = \lim_{n\to\infty} M_n$ *exists a.s. and in* L_E^q *for every* $0 < q < p$, *and* $\mathcal{L}(M) = \mu$.

EXAMPLE: Symmetric semistable measures.

We recall that an infinitely divisible measure μ on E is said to be a (r,p)-semistable probability measure $(0 < r < 1, 0 < p < 2)$ if

$$\mu^{*r} = (r^{1/p} \circ \mu) * \delta_{x_0} \quad \text{for some } x_0 \in E .$$

Here, the measure $a \circ \mu$ is defined by $(a \circ \mu)(B) = \mu(a^{-1}B)$, $B \in \mathcal{B}_E$, $a \neq 0$. The spectral representation of characteristic function of semistable measures was obtained independently by Krakowiak [9] and by Rajput and Rama-Murthy [16], which, in the symmetric case, reduces to the following:

$$(4.6) \qquad \hat{\mu}(x') = \exp\{\sum_{n=-\infty}^{\infty} r^{-n} \int_{\Delta} [\cos(r^{n/p}\langle x',x\rangle) - 1]\sigma(dx)\} ,$$

where σ is a finite symmetric measure on $\Delta = \{x \in E: r^{1/p} < \|x\| \leq 1\}$. Since

$$\hat{\mu}(x') = \exp\{\int_{\Delta}\int_{(0,\infty)} [\cos\langle x',tx\rangle - 1]\nu(dt)\sigma(dx)\} ,$$

where ν is a discrete measure concentrated on the set $\{r^{n/p}; n \in \mathbb{Z}\}$ such that $\nu(\{r^{n/p}\}) = r^{-n}$, $n \in \mathbb{Z}$, we conclude that (4.4) is satisfied with $\lambda(dx) = \sigma^{-1}(\Delta)\sigma(dx)$ and $\rho(dt) = \sigma(\Delta)\nu(dt)$. Now by elementary computations we obtain

$$R(u) = [(1/r - 1)\sigma^{-1}(\Delta)u]_r^{-1/p} ,$$

where $[t]_r = r^k$ if $r^k \leq t < r^{k-1}$. In view of Corollary 4.3(v) we get that

$$(4.7) \qquad \sum_{j=1}^{n} \varepsilon_j[(1/r - 1)\sigma^{-1}(\Delta)\gamma_j]_r^{-1/p}\, \xi_j \to S \quad \text{a.s.}$$

and in L_E^q, for every $0 \leq q < p$, and $\mathcal{L}(S) = \mu$. We have obtained a series representation of semistable random vectors in the symmetric case.

Now we note that the multipliers in (4.7) are bounded both sides, up to a constant multiplier, by $\sigma^{1/p}(\Delta)\gamma_j^{-1/p}$, because $rt < [t]_r \leq t$, $t > 0$. Further,

a p-stable limit is obtain in (4.7) when one replaces $[(1/r-1)c^{-1}(\Delta)\gamma_j]_r$ by $(1/r-1)c^{-1}(\Delta)\gamma_j$. This, in conjunction with the contraction principle, explains why the moment properties of stable and semistable distributions are so closely related. Using a different method of stochastic integeral this observation was also justified in Rosinski [20] p. 67-68 and comparisons of moments of stable and semistable measures were given.

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